

PVD Back-Analysis and Forecast Report

Calibration of consolidation parameters against field monitoring, and the forecast that follows from it

DECISION

Fit to 26 readings. Forecast settlement **928–1,432 mm** at day 1095. Record length 72 days.

Design range: the record and the laboratory uncertainty typed; point forecast 1,163 mm. 3 instruments (named in section 2).

SHORT RECORD

The record runs to day 72, below the 200-day threshold this report sets, and reaches about 66 % of the predicted final settlement. Treat the forecast as provisional and re-run as readings arrive.

Readings / parameters / dof	26 / 8 / 18	Residual multiple	×1.3 the declared noise ($\chi^2_v = 1.79$)
Typed walls	the reported set lies inside every wall — Table 5	Vacuum	off
Creep	on (L1 0.040, L2 0.040)	Smear	$k_h/k_s = 2.00$, $r_s/r_w = 3.02$

Design to the range, not to the point: the range is what the record and the typed uncertainty support together.

Project	Riverside Embankment — Phase 2 back-analysis	Job no.	EX-2026-004
Client	Example Infrastructure Authority	Document no.	EX-GEO-CAL-013
Site	Example site, VIC	Revision	A
Analysis	Back-analysis — Sweep Search inversion	Date of issue	2026-10-05
Purpose of issue	Example report — for illustration	Status	EXAMPLE
Report serial	not applicable — published example		
Prepared by	A. Engineer	Checked by	B. Checker
Approved by	C. Approver	Signature / date	
PROVENANCE PVDCal — PVD Design Tool, a GeotechLogic product · build 2026-10-05 19:13:32 · solver solveAxisym2D (radial) · Latin-hypercube seed the inversion is deterministic — the same project on the same build reproduces every figure; only the order of rows in the search log follows the parallel completion order · samples 100 retained parameter sets from the Sweep Search · units m, kPa, days; mm for drain geometry, spacing and settlement			

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1. Introduction and scope

This report documents a **back-analysis** of the consolidation properties of PVD-improved ground, and the settlement forecast that follows from it. The monitoring record of 2 settlement plates and 1 piezometer was inverted for the soil parameters that reproduce it; those parameters were then carried forward to day 1095.

Instruments inverted	2 settlement plates and 1 piezometer
Observations used	26
Calibration window	day 0 to day 72 (T^* , the last reading)
Forecast horizon	day 1095 — 1023 days beyond T^*, which is extrapolation
Declared measurement noise	$\sigma_{\text{settle}} = 5.0 \text{ mm}$, $\sigma_{\text{pwp}} = 2.0 \text{ kPa}$
Forward kernel	radial only (solveAxisym2D)

What this report is for. It records what was fitted, how well it fits, and which range to carry into the next design step. It is a calculation record, not a substitute for judgement: the forecast is only as good as the record it was calibrated against, and Section 5 states the limits that apply.

2. Basis of analysis

2.1 Soil profile as back-calculated

Table 1 — Compression magnitudes and stress state of the winning parameter set. These are the values the forecast was computed with.

Layer	H (m)	γ_{sat} (kN/m ³)	e_0	C_c	C_r	σ'_p (kPa)	C_α/C_c
L1	5.00	16.0	1.60	0.390	0.025	39	0.040
L2	5.00	15.5	1.90	0.571	0.051	61	0.040

Table 2 — Flow and stiffness properties of the same parameter set. c_v governs vertical drainage; where the run is radial only it is not used and is printed as such rather than as a number.

Layer	k_0 (m/s)	C_k	β	c_v (m ² /day)	E_u (kPa)
L1	1.46e-9	0.800	5.0	not used (radial only)	4,000
L2	6.89e-10	0.950	5.0	not used (radial only)	6,000

2.2 Drain, unit cell and loading

Equivalent drain radius r_w	33.1 mm
Radius of influence r_e	508 mm
Smear	$k_h/k_s = 2.00$, $r_s/r_w = 3.02$ ($r_s = 100 \text{ mm}$, transition $r_t = 175 \text{ mm}$)
Well resistance	not part of the inversion: the drain is an ideal sink (q_w unlimited)
Vacuum / creep	vacuum off; creep on (C_α/C_c : L1 0.040, L2 0.040)
Vertical mesh Δz	0.50 m
Boundaries	top drained, base undrained

Table 3 — Surcharge loading applied during the monitored period, stage by stage.

Stage	Start (d)	Ramp (d)	Δp (kPa)	Cumulative (kPa)
1	0.0	30.0	100.0	100.0

2.3 Observation inventory

Table 4 — Every instrument used in the inversion, what it measures, how many readings it contributed, over what dates, and the measurement noise declared for it. The χ^2 of Section 4 is the sum over exactly these points.

Instrument	Position	Measures	n	First (d)	Last (d)	σ declared
Settlement plate 1	z = 0.00 m	settlement (mm)	9	5	72	5.0 mm
Settlement plate 2	z = 5.00 m	settlement (mm)	9	5	72	5.0 mm
Piezometer 1	r = 300 mm, z = 7.50 m	excess pore pressure (kPa)	8	30	72	2.0 kPa

2.4 What was fitted, and what was held

Table 5 — Every parameter the search could move, the window it was allowed, and the value it finished at; and every parameter held fixed, which contributes no width to the ranges in Section 4. The degrees of freedom in Section 4 charge one parameter for every row marked free here and nothing for a held row, because the per-layer fine-tune moves every free coordinate. A row marked **extended** finished OUTSIDE the Min/Max typed for it (by the factor shown): the search left the typed walls for that parameter, and Section 4 says whether that answer or the one inside the walls is reported. A row marked **bounded** finished on the wall of its search window (or on the C_d/C_r ratio cap): that value is a bound, not a calibrated value.

Parameter	Layer	Typed start	Search window	Free or held	Winner
k_0	L1	8.00e-10	8.00e-11 – 1.52e-9	free	1.46e-9
C_k	L1	0.800	held at the typed value	held	0.800
C_c	L1	0.500	0.188 – 1.207	free	0.390
C_r	L1	0.050	0.005 – 0.320	free	0.025
σ'_p	L1	45	12 – 147	free	39
β	L1	5.000	held at the typed value	held	5.000
C_d/C_c	L1	0.040	held at the typed value	held	0.040
k_0	L2	4.80e-10	4.80e-11 – 9.12e-10	free	6.89e-10
C_k	L2	0.950	held at the typed value	held	0.950
C_c	L2	0.700	0.264 – 1.690	free	0.571
C_r	L2	0.070	0.007 – 0.448	free	0.051
σ'_p	L2	70	18 – 229	free	61
β	L2	5.000	held at the typed value	held	5.000
C_d/C_c	L2	0.040	held at the typed value	held	0.040
η	all layers	2.000	held at the typed value	held	2.000
r_s	all layers	100	held at the typed value	held	100
r_t	all layers	175	held at the typed value	held	175

2.5 In-situ effective stress and over-consolidation

Table 6 — Effective stress at each layer mid-depth from the winning set's own profile: $\sigma'_0(z) = \Sigma(\gamma - \gamma_w) \cdot \Delta z$ below the water table, the typed and the back-calculated σ'_p , the value the solver used after its floor at σ'_0 , and the resulting OCR.

Layer	z_{mid} (m)	σ'_0 (kPa)	σ'_p typed (kPa)	σ'_p back-calculated (kPa)	σ'_p used (kPa)	OCR	Interpretation
L1	2.50	15.0	45.0	38.8	38.8	2.58	over-consolidated
L2	7.50	43.8	70.0	61.4	61.4	1.40	over-consolidated

3. Analysis method

Each candidate parameter set is solved with the same forward kernel this application uses for Design and Simulate, and scored against the record by

$$\chi^2 = \sum_i [(\text{observed}_i - \text{predicted}_i) / \sigma_i]^2$$

summed over every reading in Table 4, with σ the measurement noise declared for that instrument. Two observation types were fitted with **group balance** on: each type is given one vote, $\lambda_g = (N/G)/n_g$, so the many-reading type cannot outvote the other; the χ^2 reported is that balanced value, and the raw sum is printed beside it. Dividing by the degrees of freedom — the observations less one for every parameter marked free in Table 5 — gives the reduced χ^2 , which is 1 when the residuals sit at the declared noise level.

One search track ran. The scan found a single valley, so nothing was refined in parallel with the reported answer. A second track appears only where two distinct valleys are found, and a run with one valley is correspondingly shorter.

The search. Sweep Search works in stages: a scan of the three global axes (a magnitude tilt over the compression indices, a yield-stress and β axis, and a speed axis carrying k_0 and C_k), then alternating refits of speed and magnitude, then gated per-layer fine-tuning, and finally a set of admissible parameter sets retained around the winner. Every stage optimises and reports the same χ^2 , so the numbers in Section 4 are comparable across stages.

How the ranges are formed. The retained sets are those whose fit remains consistent with the declared noise; each is weighted by its own misfit, and the reported interval is the weighted 5th to 95th percentile of the quantity in question. The *design range* additionally propagates the laboratory uncertainty you typed, and on this run it contains the χ^2 region, so it is wider than the record alone would give and is the range to design to.

4. Results

4.1 Fit quality

Table 7 — Goodness of fit of the winning parameter set. The degrees of freedom are the observations of Table 4 less one for every parameter marked free in Table 5.

χ^2 (balanced)	Observations	Parameters spent	Degrees of freedom	Reduced χ^2	Residual multiple	Retained sets	Search ended
32.1 balanced; raw 40.9	26	8	18	1.79	×1.3	100	converged

The residuals of this fit are above the declared noise by a factor of about 1.3. Section 4.5 plots them; a reduced χ^2 above 1 means either that the declared σ was tighter than the instruments really are, or that the model cannot follow the record — the residual pattern distinguishes the two, and a random scatter about zero points at the first.

4.2 Forecast, and the range to design to

Forecast settlement at day 1095 **1,163 mm**

Table 8 — The reported intervals for that settlement. One quantity per column: a low, a high and the width they span. The design range is the one to carry forward.

Interval	Low (mm)	High (mm)	Width (mm)	What it is
Design range	928	1,432	504	the record and the laboratory uncertainty you typed — design to this
Fit-narrowed	1,007	1,349	341	what the record alone supports, weighted by misfit
χ^2 region	1,100	1,227	126	the sets whose fit is statistically indistinguishable from the winner

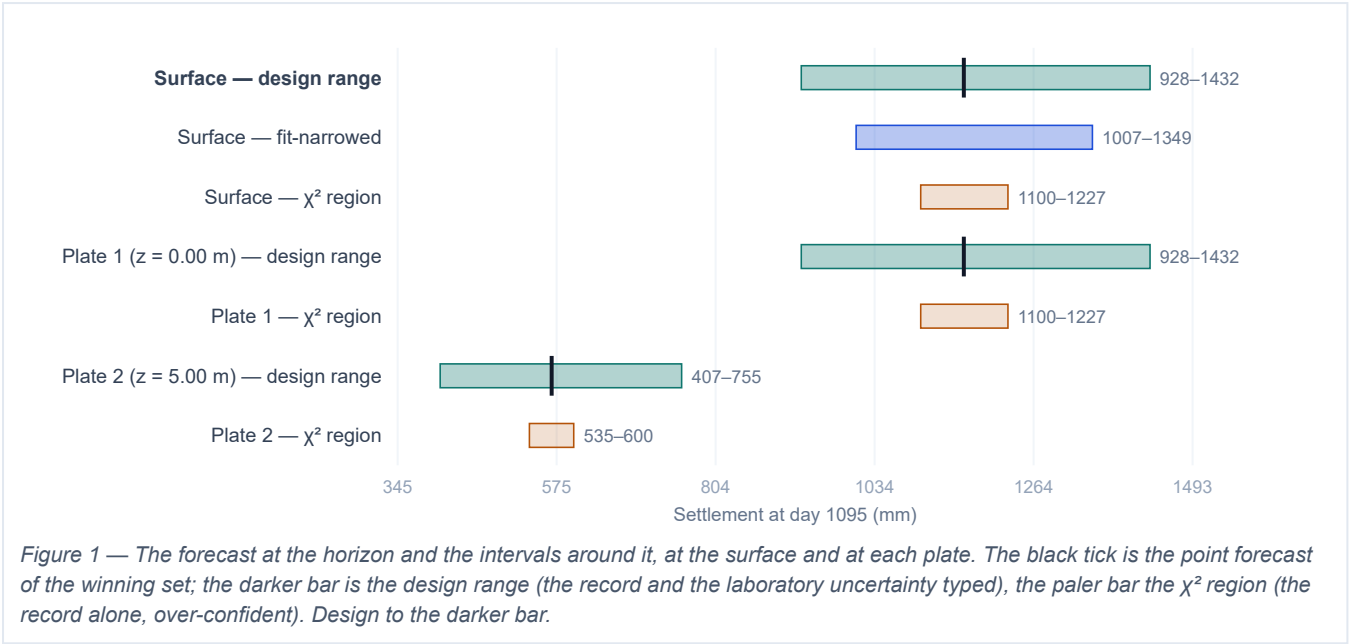
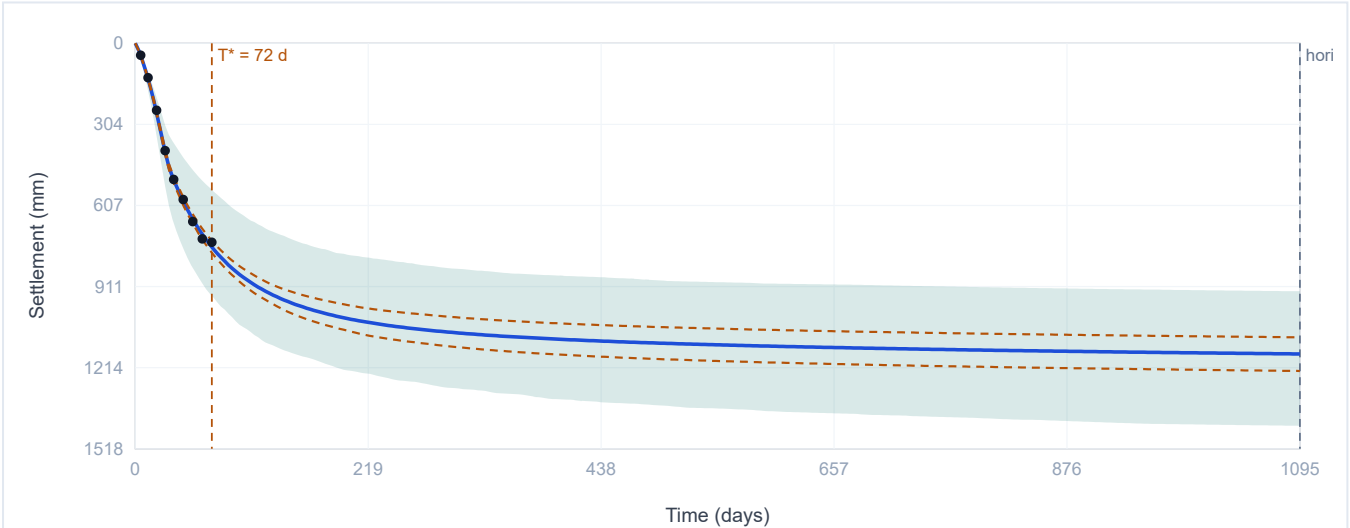


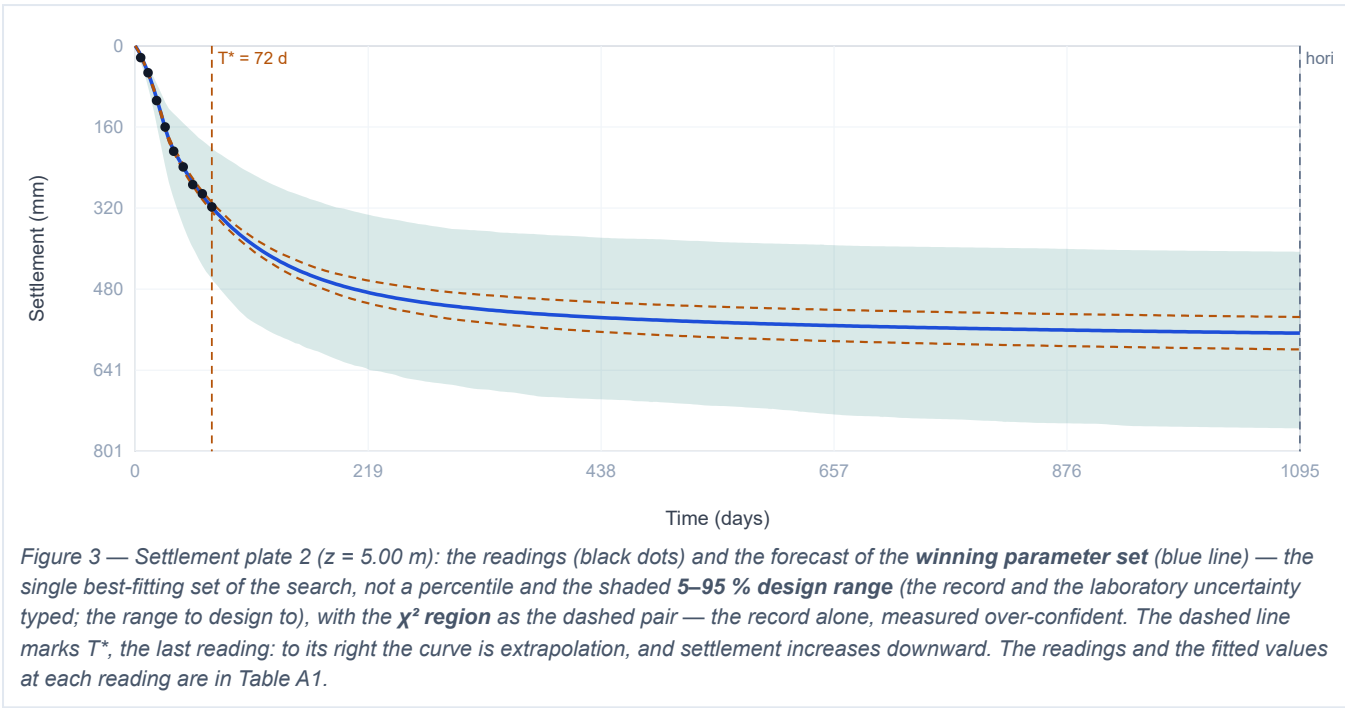
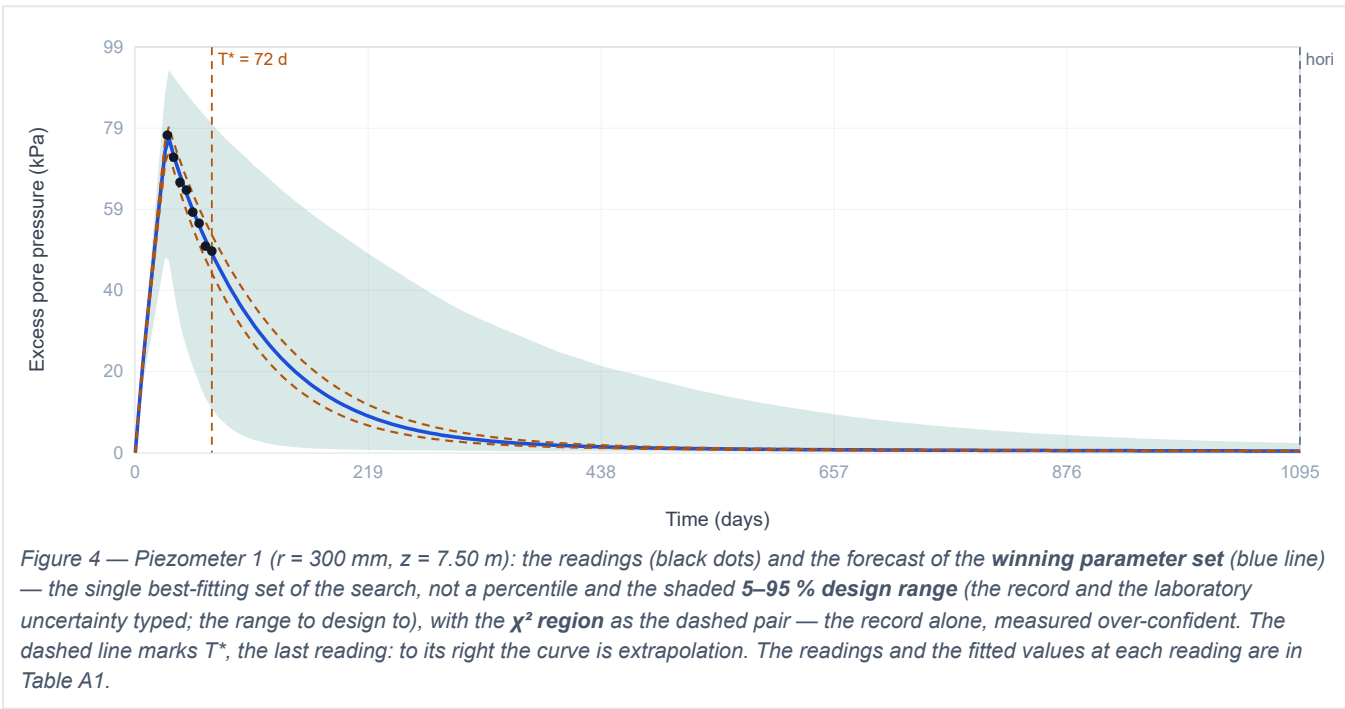
Table 9 — The forecast at each settlement plate. The headline above is the surface ($z = 0$), the settlement of the whole column; a plate at depth reads the compression of the soil below it.

Plate	z (m)	Forecast (mm)	Design range (mm)	χ^2 region (mm)
Settlement plate 1	0.00	1,163	928–1,432	1,100–1,227
Settlement plate 2	5.00	567	407–755	535–600

This forecast is an extrapolation. The parameters were calibrated against readings that stop at day 72 (T^*), and the forecast is carried to day 1095 — 1023 days beyond the last observation. Nothing in the record constrains the behaviour after T^* ; what carries it there is the model. Re-run the inversion as further readings arrive.

4.3 Instrument fits





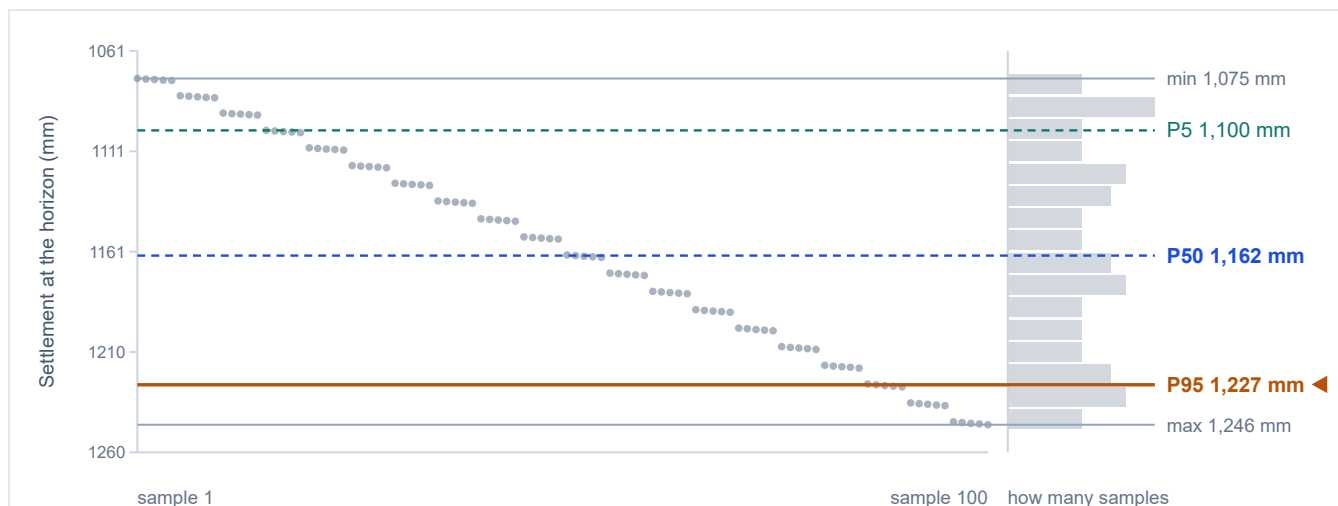


Figure 5 — Settlement plate 1 ($z = 0.00$ m): every retained parameter set's settlement at the horizon (one mark per set, grey) with the sideways histogram, and the sets at the 5th, 50th and 95th percentiles of the χ^2 region marked. The high end is the conservative one for a settlement. A bunched cloud means the record pins this plate; a wide or two-humped one means it does not, whatever the three numbers say.

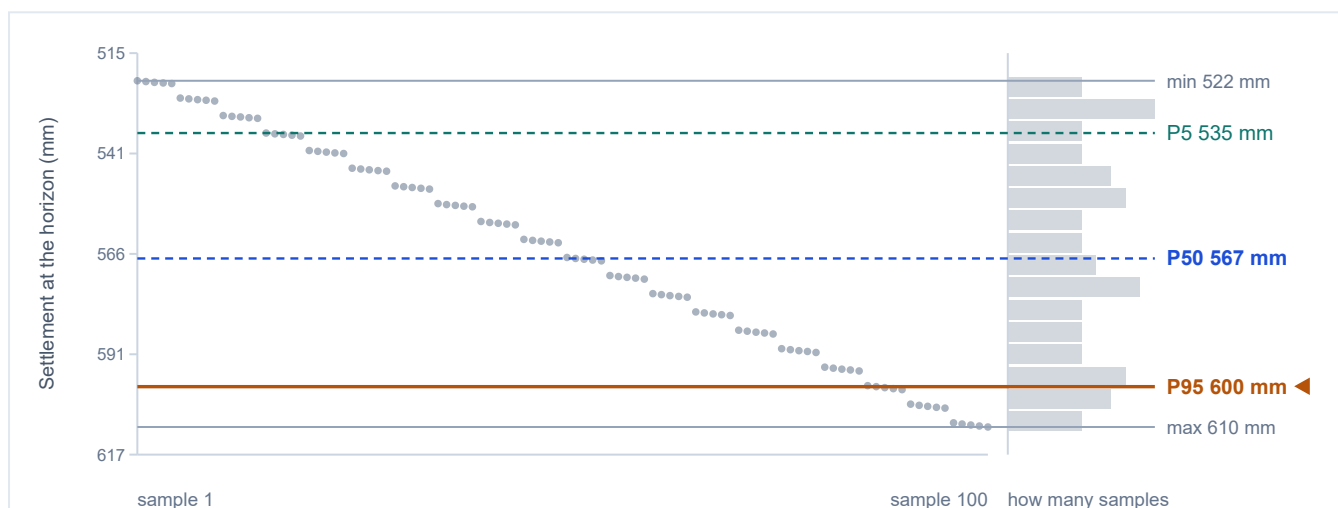


Figure 6 — Settlement plate 2 ($z = 5.00$ m): every retained parameter set's settlement at the horizon (one mark per set, grey) with the sideways histogram, and the sets at the 5th, 50th and 95th percentiles of the χ^2 region marked. The high end is the conservative one for a settlement. A bunched cloud means the record pins this plate; a wide or two-humped one means it does not, whatever the three numbers say.

4.4 Back-calculated parameters

Table 10 — Every searchable parameter by layer: the typed value, the Min–Max typed for it, and the parameter sets that produced the 5th, 50th and 95th percentile forecasts (each column is one real set, not a marginal range). The last column says where the best set stands against the typed walls: **inside**, **extended** (outside, by the factor shown) or **bounded** (on a search wall).

Layer	Parameter	Typed	Min – Max typed	Set at the χ^2 region's P5	Best (P50)	Set at the χ^2 region's P95	Against the walls
L1	k_0 (m/s)	8.00e-10	8.00e-11 – 1.52e-9	1.37e-9	1.46e-9	1.41e-9	inside
L1	C_k	0.800	held	0.800	0.800	0.800	held
L1	C_c	0.500	0.350 – 0.650	0.377	0.390	0.404	inside
L1	C_r	0.0500	0.0200 – 0.0800	0.0228	0.0247	0.0266	inside
L1	σ'_p (kPa)	45.0	27.0 – 63.0	40.6	38.8	37.0	inside

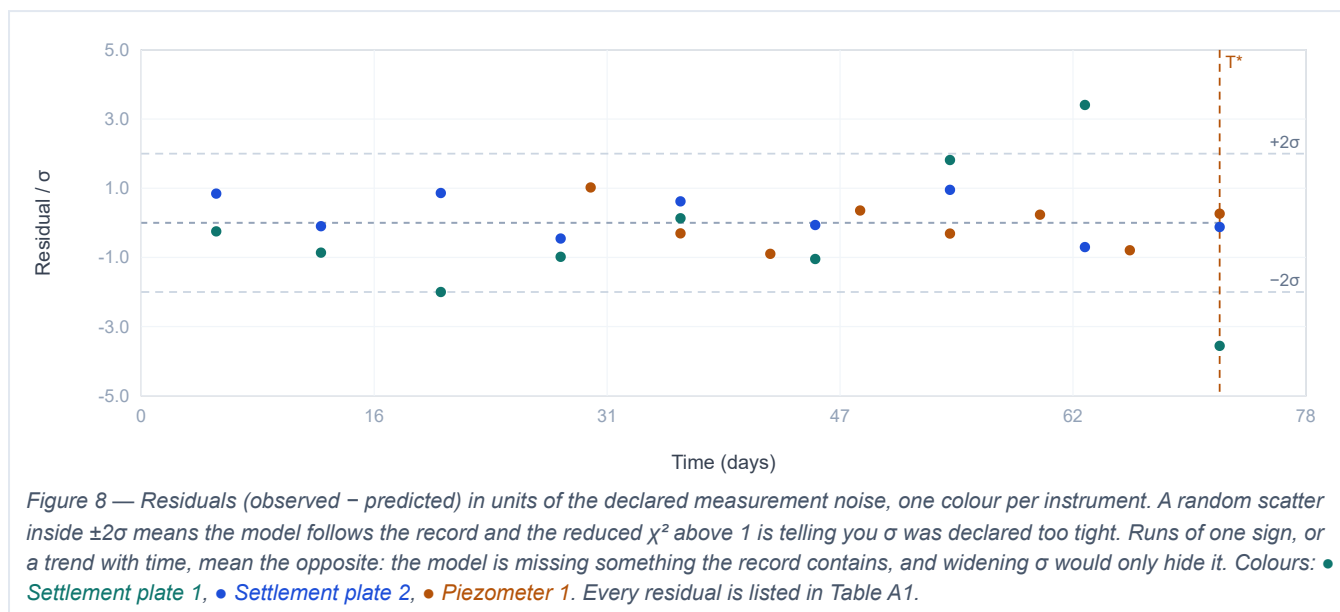
Table 10 (continued)

Layer	Parameter	Typed	Min – Max typed	Set at the χ^2 region's P5	Best (P50)	Set at the χ^2 region's P95	Against the walls
L1	β	5.00	held	5.00	5.00	5.00	held
L1	C_α/C_c	0.0400	held	0.0400	0.0400	0.0400	held
L2	k_0 (m/s)	4.80e-10	4.80e-11 – 9.12e-10	6.50e-10	6.89e-10	6.69e-10	inside
L2	C_k	0.950	held	0.950	0.950	0.950	held
L2	C_c	0.700	0.490 – 0.910	0.551	0.571	0.590	inside
L2	C_r	0.0700	0.0280 – 0.112	0.0469	0.0506	0.0546	inside
L2	σ'_p (kPa)	70.0	42.0 – 98.0	64.3	61.4	58.6	inside
L2	β	5.00	held	5.00	5.00	5.00	held
L2	C_α/C_c	0.0400	held	0.0400	0.0400	0.0400	held



Figure 7 — How the fit improved: the lowest reduced χ^2 after each cycle of the search, through 4 stages (joint fit, sweep, stage 3, simple). Values above 14.2 are clipped to the frame. A curve that flattens early and stays flat is a search that converged; one still falling at its last cycle was stopped by a rule, not by convergence.

4.5 Residuals



5. Assumptions and limitations

- The ground is idealised as a **unit cell** around one drain: one drain, its share of soil, no interaction with neighbouring cells beyond that share.
- **Vertical flow is off in the forward model.** The fitted k_0 therefore absorbs some of the missing vertical drainage path. Do not export this k_0 into a coupled model without saying so.
- **Well resistance is not part of the inversion.** The drain is treated as a perfect sink unless a discharge capacity was entered and checked; if it was, the check is reported in the Design report for the same profile.
- **Parameters held fixed add no width** to the ranges of Section 4. Table 5 lists what was held: a range computed with a parameter pinned is narrower than the ground warrants, and the honest way to widen it is to free that parameter and re-run.
- **The record is short.** The record runs to day 72, below the 200-day threshold this report sets, and reaches about 66 % of the predicted final settlement. Treat the forecast as provisional and re-run as readings arrive.
- **Everything after $T^* = 72$ d is extrapolation.** The record cannot confirm or refute the forecast beyond its last reading.
- A reduced χ^2 above 1 means the residuals exceed the declared noise. That is a statement about σ or about the model — not a licence to ignore the residual pattern of Figure 8.
- The inversion reports the parameter sets consistent with *this* record. A different record from the same site may support a different set; that is a property of the data, not a fault of the search.

6. Conclusions and recommendations

The back-analysis returns a forecast settlement of **1,163 mm** at day 1095. Take the **design range of 928–1,432 mm** to the next design step: it carries both the monitoring record and the laboratory uncertainty that was typed. The parameters are calibrated to day 72; beyond that the forecast is an extrapolation. The fit is $\chi^2_v = 1.79$ on 26 observations.

1. Design to the **design range**, not to the point forecast: the point is the most likely settlement, the range is what the evidence actually supports.
2. The residuals exceed the declared measurement noise by about 1.3 times. Read Figure 8 before deciding what to do about it: a random scatter means σ was declared too tight, while a trend or a run of one sign means the model is missing something in the record.

3. **Short record.** The record runs to day 72, below the 200-day threshold this report sets, and reaches about 66 % of the predicted final settlement. Treat the forecast as provisional and re-run as readings arrive.
4. Re-run the inversion when further readings arrive. The present forecast is carried 1023 days beyond the last observation, and each new reading shortens that extrapolation.
5. Parameters held fixed (Table 5) contribute no width to the reported ranges. If any of them is genuinely uncertain, free it and re-run before treating the range as complete.

The narrative above is generated from the headline statistics of this run. It is a checklist prompt for the signing engineer, not a signed opinion.

7. References

1. Barron, R.A. (1948). Consolidation of fine-grained soils by drain wells. Transactions ASCE, 113, 718–754. — radial consolidation about a single drain, the ideal-drain limit of this unit cell
2. Hansbo, S. (1981). Consolidation of fine-grained soils by prefabricated drains. 10th ICSMFE, Stockholm, vol. 3, pp. 677–682. — radial consolidation with smear and well resistance; the closed form the radial kernel is validated against
3. Indraratna, B., Redana, I.W. (1998). Laboratory determination of smear zone due to vertical drain installation. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 124(2), 180–184. — laboratory determination of the smear zone, the basis for the adopted r_s and η
4. Sharma, J.S., Xiao, D. (2000). Characterization of a smear zone around vertical drains by large-scale laboratory tests. Canadian Geotechnical Journal, 37, 1265–1271. — large-scale laboratory characterisation of the smear zone around a vertical drain
5. Walker, R. and Indraratna, B. (2006). Vertical drain consolidation with parabolic distribution of permeability in smear zone. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 132(7), 937–941. — permeability varying through the smear annulus, the behaviour represented here by the transition zone $r_s \rightarrow r_t$
6. Indraratna, B., Rujikiatkamjorn, C. and Sathananthan, I. (2005). Radial consolidation of clay using compressibility indices and varying horizontal permeability. Canadian Geotechnical Journal, 42(5), 1330–1341. — radial consolidation formulated on compressibility indices and varying horizontal permeability — the C_c/C_k formulation this solver uses
7. Abuel-Naga, H.M., Pender, M.J. (2012). Modified Terzaghi consolidation curves with effective stress dependent coefficient of consolidation. Géotechnique Letters, 2, 43–48. — effective-stress-dependent coefficient of consolidation, the basis for re-evaluating $c_h(\sigma')$ at every step

Project records used by this analysis:

1. Settlement plate 1 ($z = 0.00$ m) — settlement (mm), 9 readings, day 5–72, monitoring record supplied with the project
2. Settlement plate 2 ($z = 5.00$ m) — settlement (mm), 9 readings, day 5–72, monitoring record supplied with the project
3. Piezometer 1 ($r = 300$ mm, $z = 7.50$ m) — excess pore pressure (kPa), 8 readings, day 30–72, monitoring record supplied with the project

The list above is assembled from the physics this run used: smear and transition references are included because $\eta > 1$. Vacuum references are omitted because no vacuum is applied.

8. Software provenance and validation

Produced by **PVDCal — PVD Design Tool**, a GeotechLogic product, build **2026-10-05 19:13:32**. This file was computed with the **radial-only kernel (solveAxisym2D)**. The inversion drives that same forward kernel on every candidate parameter set; no simplified or surrogate solver is used inside the search.

Table 11 — The closed-form benchmark suite that ships inside the application, reproducible from its Validation page. Errors are in percentage points of the degree of consolidation. Rows marked • are the checks that govern the kernel used for this file.

Benchmark	What it tests	Mean abs. error (pp)	Max error (pp)
Olson–Barron •	ramp loading, ideal drain	0.398	0.742

Table 11 (continued)

Benchmark	What it tests	Mean abs. error (pp)	Max error (pp)
Hansbo •	radial consolidation with smear	0.299	1.030
Hansbo, layered solver •	layered stack, ideal drain	0.530	2.179
Hansbo with smear, layered solver •	layered stack, with smear	0.254	0.963
Terzaghi	pure vertical drainage	0.116	1.119
Carrillo	radial and vertical combined	0.000	0.001
Vacuum-assisted	suction boundary condition	0.530	2.178
Partial penetration	limits: full $\equiv$ Hansbo, none $\equiv$ Terzaghi	0.016	0.290

Spatial convergence is second order in both directions: 0.021 percentage points of U at 80 radial cells against a 640-cell reference, and 0.120 percentage points RMS at the default $\Delta z = 0.5$ m against the exact Terzaghi series. The discretisation, its measured convergence behaviour and the full benchmark set are documented in the accompanying *Numerical Method Guide*, and every benchmark above can be re-run from inside the application rather than taken from this page.

Responsibility. These results depend on the input parameters tabulated in Section 2 and on the assumptions and limitations in Section 5. The narrative in Section 6 is generated automatically from the headline statistics and the configuration of this run: it is a checklist prompt, not a signed opinion. The software is a calculation aid; responsibility for the design rests with the signing engineer.

Appendix A. Readings, fitted values and residuals

Every point on the instrument and residual figures of Section 4. Settlement in mm, pore pressure in kPa; the fitted value is the winning parameter set at the reading's own date, and the residual is (observed – fitted)/ σ with σ from Table 4.

Table A1 — Readings against the fit, instrument by instrument.

Instrument	Day	Observed	Fitted	Band 5 %	Band 95 %	Residual / σ
Settlement plate 1	5	45	46	45	48	-0.25
Settlement plate 1	12	129	133	130	137	-0.86
Settlement plate 1	20	251	261	254	267	-2.00
Settlement plate 1	28	402	407	397	417	-0.98
Settlement plate 1	36	510	509	496	522	0.13
Settlement plate 1	45	585	590	575	605	-1.05
Settlement plate 1	54	667	658	640	674	1.82
Settlement plate 1	63	732	715	694	734	3.41
Settlement plate 1	72	745	763	740	784	-3.56
Settlement plate 2	5	23	19	18	20	0.85
Settlement plate 2	12	53	53	51	56	-0.10
Settlement plate 2	20	108	104	100	107	0.86
Settlement plate 2	28	160	162	158	166	-0.45
Settlement plate 2	36	208	205	200	210	0.62
Settlement plate 2	45	239	239	233	246	-0.06
Settlement plate 2	54	274	269	262	276	0.95
Settlement plate 2	63	292	296	287	303	-0.70
Settlement plate 2	72	318	319	309	327	-0.12
Piezometer 1	30	77.5	75.5	72.4	78.1	1.02
Piezometer 1	36	72.1	72.7	69.1	75.9	-0.31
Piezometer 1	42	66.0	67.8	63.8	71.3	-0.90
Piezometer 1	48	64.1	63.4	59.1	67.1	0.36
Piezometer 1	54	58.7	59.3	54.9	63.3	-0.31
Piezometer 1	60	56.0	55.5	51.0	59.7	0.23
Piezometer 1	66	50.4	52.0	47.3	56.3	-0.80
Piezometer 1	72	49.2	48.7	43.9	53.2	0.27